Neural Potential Field for Obstacle-Aware Local Motion Planning

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Abstract-Model predictive control (MPC) may provide local motion planning for mobile robotic platforms. The challenging aspect is the analytic representation of collision cost for the case when both the obstacle map and robot footprint are arbitrary. We propose a Neural Potential Field: a neural network model that returns a differentiable collision cost based on robot pose, obstacle map, and robot footprint. The differentiability of our model allows its usage within the MPC solver. It is computationally hard to solve problems with a very high number of parameters. Therefore, our architecture includes neural image encoders, which transform obstacle maps and robot footprints into embeddings, which reduce problem dimensionality by two orders of magnitude. The reference data for network training are generated based on algorithmic calculation of a signed distance function. Comparative experiments showed that the proposed approach is comparable with existing local planners: it provides trajectories with outperforming smoothness, comparable path length, and safe distance from obstacles.

I. INTRODUCTION

Obstacle-aware motion planning is essential for autonomous mobile robots. Various methods may solve this task, including numerical optimization, especially nonlinear Model Predictive Control (MPC) [1–7]. Optimization allows the planner to transform a rough global path into a smooth trajectory, taking into account obstacles and kinodynamic constraints of the robot.

Obstacle avoidance may be inserted into trajectory optimization as a penalty term in the cost function (e.g., [4, 8]). The penalty function forms a repulsive Artificial Potential Field (APF); its gradient directs toward the safer solution [9]. This allows the controller to find a proper balance between the safety of the trajectory and its similarity to the reference path. Therefore, the function which forms the repulsive APF should be differentiable. The values of the repulsive APF may be easily computed based on the signed distance function (SDF) from the robot to the nearest obstacle point on the map. However, SDF is computed by specific algorithms. It is not a differentiable function for the general case. It is easy to define it analytically when two requirements are satisfied: first, the robot is pointwise or circular, and second, the obstacles have known simple geometric shapes. If both the robot footprint and obstacle map are arbitrary, finding a differentiable approximation of the SDF [8, 10] is hard.

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 $\boldsymbol{x} = (x, y, \theta)^T$ Positional E_{γ} Encoder Obstacle map -------Neural E_{map} Map Jobst Potential Encode Function Robot footprint E_{fp} Footprint Learning (NPField) _ _ _ $x(0), x_{ref}(1...m)$ Inference (Controller) Obstacle map **x**(0), $\mathbf{x}_{ref}(1 \dots m),$ Numerio $x_{opt}(1 \dots m)$ E_{map}, E_{fp} Robot footprint

Fig. 1. Common scheme of the proposed approach. Our controller (bottom half of the figure) consists of a parameter definition module and an MPC solver, which optimize the trajectory for a defined set of parameters. Our common neural architecture (NPField, top half of the figure) consists of image encoders and a neural potential function (NPFunction). We train NPFunction to predict the obstacle-repulsive potential for a given robot pose and given embeddings of the obstacle map and the robot footprint. The trained neural potential function is used for trajectory optimization within the MPC solver. Map and footprint encoders are removed from the optimization loop to decrease the dimensionality of the MPC problem. They are used for data preparation as both map and footprint are considered constant within the prediction horizon. More precise schemes of this architecture are given in figures 2 (Controller) and 3 (Network).

We propose a Neural Potential Field (NPField) - a neural network for calculating artificial potential. Our idea is conceptually inspired by the NeRF (Neural Radiance Field) model [11], which takes the position and orientation of the camera as an input and returns image intensity as an output. Our model takes the position and orientation of the robot together with the obstacle map and robot footprint as input and returns the value of repulsive potential as an output. We aim not to obtain this values themselves but to use the trained model within the optimization loop. There are several works where neural networks provide discretized costmaps in which values are used for search-based [12] or sampling-based planning [13, 14]. Our goal instead is to provide continuously differentiable function, which gradient is helpful for optimization. The key conceptual scheme of our approach is shown in Fig. 1.

MPC solvers are sensitive to the number of input parameters: a high value leads to a drastic increase in computational expenses. We use image encoding within our architecture

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to reduce the number of parameters by replacing maps and footprints with their more compact embeddings. The top part of the fig. 1 presents our neural network architecture, while the bottom part shows the data flow of our controller. The neural network consists of two main parts: encoder block and Neural Potential Function (NPFunction) - a subnetwork that calculates the potential for a single robot configuration. Our controller consists of two high-level blocks. The first block includes the algorithms, which define parameters for the MPC problem based on actual sensor data. Such a definition is made once per each iteration of the control loop. The second block includes a numerical solver for the control problem, which iteratively optimizes the trajectory based on the pre-defined parameters. NPField is trained as a single architecture and then divided into two parts. Image encoders are inserted into the parameter definition block, while NPFunction is integrated into the numerical MPC solver. For each controller step, encoders are called once, while NPFunction is called and differentiated multiple times within the optimization procedure.

This work mainly contributes in the following aspects:

- Novel architecture for MPC local planner, where the neural model estimates collision cost.
- Novel neural architecture for calculating APF based on the obstacle map, robot pose, and footprint.
- An approach for generating the dataset for training the neural model.

The last subsection of the next section provides a discussion on the place of our approach among the others.

II. RELATED WORKS

A. Planning

Planning task may be solved by various methods, which could be categorized into the following main groups (see review [15]): search-based planning (most of these methods are based on A* graph search algorithm [16, 17]), sampling-based planning (most of these methods are based either on Rapidly-exploring Random Trees [18] or on Probabilistic RoadMaps [19]), motion primitives [20], reinforcement learning [21] and trajectory optimization. We consider optimization-based planning in this letter.

We can define two statements of planning tasks – global path planning (define a reference of intermediate robot configurations based on given initial and destination configurations) and local motion planning (define a smooth and safe trajectory based on a given part of the global plan). The artificial potential field was initially proposed [9] for global planning: the robot's path is obtained as a trajectory of the gradient descent in the potential field from the starting point towards the destination point. This planning approach can easily stuck in the local minimum. Therefore, it is less popular than A*, RRT, or PRM. However, it is still useful [22–24]. To avoid stucking in a local minimum, trajectory optimization is often done locally together with global planners [1, 7]. Global planner generates a rough suboptimal path, which is then optimized.

B. Obstacle models for trajectory optimization

Collision detection itself is considered in many works, e.g., [25–27]; we are now interested in analytical models suitable for trajectory optimization. The safe path may be guaranteed using convex approximations of the free space [1, 10]. The disadvantage of such an approximation is that the free space outside the approximated region is prohibited. Alternatively, obstacles may be approximated instead of free space [4–7]. Interception of the trajectory with the borderlines of the approximated regions may be modeled within the MPC solver. The approaches above require modeling either free space or obstacles as simple geometric shapes, such as points [4], circles [1, 5], polygons [6, 7], or polylines [28]. The question of how to obtain this representation from the common obstacle map is often not considered.

In the case when it is impossible or too complicated to provide differentiable collision models, one can use less stable techniques based on numerical gradients [8], stochastic gradients [29] or gradient-free sampling-based optimization [30, 31]. We consider another option, where a neural model approximates the repulsive potential. A number of works exist [32] on learning Control Barrier Functions for ensuring the safety of mobile systems such as drones [33] and cars [34] within the controller. The work [35] provides differentiable collision distance estimation for a 2D manipulator based on a graph neural network. [36] use the loss function of the network as a collision penalty: the trajectory is optimized during the network training for the fixed obstacle map.

C. Neural Models within MPC Optimization

Integration of neural models into the MPC control loop was considered in a number of tasks. The challenging aspect here is the high computational cost of deep neural models. Accurate deep models by [37, 38] were not real-time and were presented as a proof of concept. Real-time inference may be achieved by significantly reducing model capacity [38]. Approaches [39-41] insert lightweight network into realtime MPC control loop. [42] achieve use of the relatively large neural model within real-time Acados MPC solver [43] by introducing ML-CasADI framework [44]. Experiments by [42] showed that direct insertion of the neural model into MPC-solver is effective for the networks with up to 50,000 parameters. Most works above use neural networks for approximating process dynamics. On the contrary, we are interested in approximating obstacle-related cost terms and control barrier functions [32-34]. In [45], a neural network was used to update a cost function for manipulator visual servoing. Its architecture includes a neural encoder for camera images and a cost-update network for quadratic programming. There is also a set of works where a neural network was applied for choosing weighting factors for various terms of the cost function, e.g., [46, 47].

D. Place of our approach among the others

We propose a neural model for estimating repulsive potential, which has the set of properties listed in table I. This table also include some related works, which partly satisfy

Property	[36]	[45]	[33]	[34]	Ours
Provide obstacle avoidance for mo- bile robot	\checkmark		\checkmark	\checkmark	\checkmark
Provide differentiable potential function	~		\checkmark	√	✓
Exploit the capacity of deep neural models with 50K+ parameters		√	\checkmark	√	√
Reproduces obstacle maps with complicated, non-convex structures and allows for optimization of long trajectories within this map	√		√		√
Work within realtime MPC Planner		\checkmark		\checkmark	\checkmark
Single model for multiple maps		\checkmark			\checkmark

TABLE I

COMPARISON OF OUR APPROACH WITH MOST SIMILAR EXISTING WORKS

these properties. Other mentioned works satisfy less number of properties. Unique additional property of our approach is encoding the footprint of the mobile robot. It allows one to use a single model for mobile robots with different shapes. Note that in this work, we only prove a concept of footprint encoding: our training set consists of samples with two various footprints, and we show that the networks learn their collision model. We consider the deeper study of footprint learning (including footprint generalization) to be a part of future work.

III. CONTROL APPROACH

A. MPC Statement

Local trajectory optimization may be formulated as a nonlinear model predictive control problem with continuous dynamics and discrete control:

$$\{\mathbf{x}_{opt}[i], \mathbf{u}_{opt}[i]\}_{i=k}^{k+m} = \arg\min\sum_{i=k}^{k+m} J(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]), \quad (1a)$$

s.t.

$$\frac{dx_1[i]}{dt} = f_1(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]),$$

$$\frac{dx_2[i]}{dt} = f_2(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]),$$
(1b)

. . .

$$\begin{aligned} \frac{dx_{\mu}[i]}{dt} &= f_{\mu}(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]), \\ h_{1}(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]) &\leq 0, \\ h_{2}(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]) &\leq 0, \\ & \dots \\ h_{\chi}(\mathbf{x}[i], \mathbf{u}[i], \mathbf{p}[i]) &\leq 0. \end{aligned}$$
(1c)

Here p is prediction horizon, $\mathbf{x}[i]$ is μ -size state vector (at the beginning of step [i]), $\mathbf{u}[i]$ is ν -size control vector (constant within step i) $\mathbf{p}[i]$ is κ -vector of process parameters (relevant for the step i). (1a) specify the cost function J: a sum of functions J[i], which are calculated for each node. (1b) define continuous dynamics of the process. (1c) is a set of inequality constraints which must be satisfied within the whole process. Optimization procedure aims to find the reference of $\{\mathbf{x}_{opt}[i], \mathbf{u}_{opt}[i]\}_{i=k}^{k+m}$ that provide minimum J. The view of the equation (1b) depend on the construction of the mobile robot. In this work we consider a differential drive model and a bicycle model (used in numerical experiments). Differential drive model is specified as follows:

$$\frac{dx}{dt} = v\cos\theta,$$

$$\frac{dy}{dt} = v\sin\theta,$$

$$\frac{dv}{dt} = a,$$

$$\frac{d\theta}{dt} = \omega.$$
(2)

State vector $\mathbf{x} = (x, y, v, \theta)^T$ include Cartesian position of the robot x, y, its linear velocity v, and its orientation θ . Control vector $\mathbf{u} = (a, \omega)^T$ include linear acceleration a and angular velocity ω . For bicycle model $\mathbf{u} = (a, \delta)^T$ where δ is steering angle. Derivative of θ is calculated as $d\theta/dt = v/L*$ tan θ where L is base length of the robot. Other derivatives are defined like in (2).

For the optimal control problem of this model, the following cost function is introduced:

$$J[i] = J_s(\mathbf{x}[i], \mathbf{u}[i], \mathbf{x}_r[i]) + J_o(\mathbf{x}[i], \mathbf{p}_o[i]).$$
(3)

 J_s term enforce the trajectory to follow the reference values \mathbf{x}_r from the global plan, while J_o term push the trajectory farther from obstacles. $\mathbf{p}_o[i]$ is a vector of obstacle-related parameters. Whole parameter vector for the system (1) is $\mathbf{p}[i] = ((\mathbf{x}_r[i])^T, (\mathbf{p}_o[i])^T)^T$. In our approach, the neural network is applied to compute J_o while J_s is calculated as follows:

$$J_{s}[i] = \sum_{j=1}^{\mu} w_{xj} (x_{j}[i] - x_{j(ref)}[i])^{2} + \sum_{k=1}^{\nu} w_{uk} u_{k}^{2}[i]$$
(4)

Here w_{xj} , w_{uk} are the weights of the respective terms, $x_{j(ref)}[i]$ is a reference value of the respective state (taken from the global plan).

Constraint equation (1c) include only constraints on minimum and maximum values of the separate variables.

B. Control architecture

An architecture of our controller is shown in Fig. 2.Solution of the problem (1) is obtained iteratively using Sequential Quadratic Programming (*optimization loop* in Fig. 2). MPC controller uses the solution to update the trajectory online (*control loop* in Fig. 2). At the timestep k it optimizes the trajectory for the next m steps, and then the first optimized control input \mathbf{u}_k is sent to the robot. After that optimization is repeated for the steps from k+1 to k+m+1.

MPC-solver is intended to provide solution of the problem (1) with (4) as (1a) and (2) as (1b). During optimization, it communicates with integrated NPFunction, which provides values and gradients of J_{obst} .

The objective of the neural network is to project the robot's footprint, obstacle map, and robot poses onto a differentiable obstacle-repulsive potential surface. Consequently, for each coordinate within the range of the map, the neural network

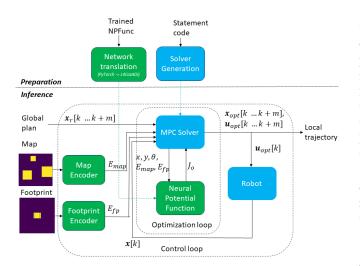


Fig. 2. Proposed architecture of the controller. Image encoders work outside the optimization loop and we just need to run them once before each optimization procedure. NPFunction works within the optimization loop and the solver uses its gradients to find a safer trajectory.

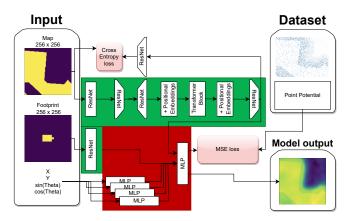


Fig. 3. Proposed architecture of the neural network. The green component represents the robot footprint and map encoder, which generates embeddings that are consistent for all coordinates within the map. The red component signifies the final coordinate potential predictor, which contains an order of magnitude fewer parameters.

outputs a corresponding potential value. To ensure computational feasibility, we partitioned the neural network into two blocks: first, encoders, and, second, potential predictor. Encoders compress high-dimensional maps into a compact representation, thereby enabling the computation of Jacobian and Hessian matrices of the control problem within the solver. Encoders work outside the optimization loop: provided embeddings E_{map} and E_{fp} are sent to the solver as obstacle-related problem parameters \mathbf{p}_o . We assume the local map and robot footprint to be fixed within the prediction horizon. While the robot is following the global plan, the local map slides according to its current position.

IV. NEURAL POTENTIAL FIELD

A. Network architecture

Proposed neural architecture Fig. 3 consists of three primary components: a ResNet Encoder, a Spatial Transformer, and a ResNet Decoder. ResNet blocks are used with the objective of extracting local features from the obstacle map which contains a lot of corners and narrow passages. The Spatial Transformer utilizes the self-attention mechanism [48] to establish global relations among these features, assessing the significance of one feature in relation to others. Consequently, we employ the positional embedding technique from Visual Transformers [49]. Lastly, the ResNet Decoder processes the transformed feature maps to generate the final output.

To mitigate the model's tendency to truncate critical details of the obstacle map necessary for navigation, we incorporated a map reconstruction loss based on Cross-Entropy. For the predictions of potential points, we employed the Mean Squared Error (MSE) loss.

B. Training data

One unit of the training set include $\{I_{map}, I_{fp}, x, y, \theta, J_o\}$, where I_{map} and I_{fp} are 2D images of the obstacle map and the robot footprint. The dataset should include various samples of robot positions from various maps. The maps are cropped from the MovingAI planning dataset [50]. For each map, we generate a set of random robot poses and calculate reference values for them using the following algorithm.

- 1) Obstacle map is transformed into a costmap:
 - a) Signed distance function (SDF) is calculated algorithmically for each cell on the map. SDF is equal to the distance from the current cell to the nearest obstacle border. It is positive for free space cells and negative for obstacle cells.
 - b) Repulsive potential is calculated for each cell: $J_o = w_1(\pi/2 + \arctan(w_2 - w_2SDF))$. This is a sigmoid function, which is low far from obstacles, asymptotically strives to w_1 inside obstacles, and has maximum derivative on the obstacle border.
- 2) Repulsive potential is calculated for each random pose of the robot within the submap. For this purpose robot's footprint is projected onto the map according to the pose. The maximum potential among the footprintcovered cells is chosen as a repulsive potential.

A pivotal aspect of the training process was the dataset sampling strategy. Utilizing a random sampling strategy across the map led to the network overfitting to larger values and disregarding narrow passages. This is because of the walls, which are statistically overwhelming compared to free space, but are irrelevant for navigation as we explicitly avoid planning through obstacles. To address this, we modified the sampling strategy such that 80% of points are sampled with intermediate potential values.

V. IMPLEMENTATION AND EXPERIMENTS

A. Implementation and performance

We implement our MPC solver with Acados framework [43], which relies on a more low-level CasADi framework [51]. We use the L4CasADi library [52] to provide optimization over NPFunction. Our local planner works together with

Theta* [53] global planner, which generates global plans as polylines. Note that Theta* uses a simplified version of the robot footprint (a circle with a diameter equal to the robot width) as it fails to provide a safe path with a complete footprint model. This simplified model does not guarantee the safety of the global plan, therefore the safety of the trajectory is provided by our local planner.

We consider obstacle maps to have a 256×256 resolution, where each pixel corresponds to 2×2 centimeters of the real environments (i.e. size of the map is 5.12×5.12 meters). We collected a dataset based on the MovingAI [50] city maps. It includes 4,000,000 samples taken from 200 maps with 2 footprints. Both footprints correspond to a real Husky UGV mobile manipulator with a UR5 robotic arm. The first one is with a folded arm, the second one is with an outstretched arm. 10,000 random poses of the robot were generated for each map with each footprint. Weighting coefficients for reference potential were set to $w_1 = 15$ and $w_2 = 10$, while the prediction horizon was set to p = 30. Dataset generation took 40 hours on the Intel Core i5-10400F CPU.

Our neural network consists of 5 million parameters, with 500,000 allocated to ResNet encoders. Encoders project each (256×256) map and robot footprint into (1×4352) embeddings. The robot's pose, represented as $X, Y, sin(\theta), cos(\theta)$, is transformed into (1,32) embeddings. The model was trained over a span of 24 hours on a server equipped with a single Nvidia Tesla V100 card with 32GB of memory.

One optimization procedure in our implementation take 60-70 ms, where data encoding take around 10 ms, while Acados solution take the rest 50-60 ms. Note that Acados solver need to warmup before reatime use: first execution of the optimization procedure may take about one second; after that the solver work faster.

B. Numerical Experiments

All experiments reported in this section were 1) conducted with the bicycle model of process dynamics, and 2) conducted on the maps from the MovingAI dataset [50], which were not used for network training.

1) Illustrative example and comparison with trajectory optimizers: An example of the trajectory generated with our planner is shown in Fig. 4 on the left. A global plan in the form of a polyline is turned into a smooth and safe trajectory. Initially, the robot turns from the obstacle and deviates from the global path, then smoothly returns to it, reaching the goal position. As a proof of concept for footprint encoding, we provide the following experiment. Consider the global plan, where the robot first moves towards the flat wall, then turns and moves parallel to the wall. In this case, a robot with a folded arm turns a little later than the one with a folded arm. Such a behavior may be seen in Fig. 4, bottom. The vellow curve relates to the outstretched arm, while the green curve relates to the folded arm. This behavior shows that the model learns different properties of two footprints, which are useful for safer trajectory planning. We compared NPField trajectories with CIAO [1] trajectory optimizer, which is based on convex approximation of the free space around the

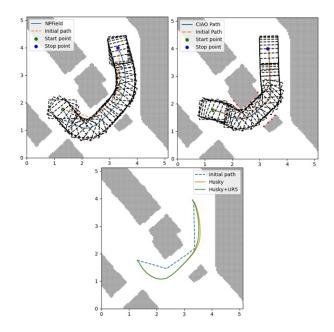


Fig. 4. Example scenario for local planning. Left: NPField trajectory. Right: CIAO trajectory. Bottom: trajectory curves for different footprints.

robot. The CIAO-generated trajectory is shown in Fig. 4 on the right. It may be seen that it keeps the robot near obstacles, nearly touching their edges. When testing on more diverse scenarios, CIAO could not find the feasible path in nearly half of the cases. It may be connected with the fact that CIAO implements collision avoidance as a set of inequality constraints, which are not differentiated during optimization. Therefore, it only checks the fact of the collision and does not balance between safety and path deviation in the cost function.

2) Comparison on BenchMR: We compare our algorithm with the baselines on 20 scenarios using BenchMR [54] framework. The tasks include moving through the narrow passages similar to those shown in Fig. 4. We compare standard metrics: planning time, path length, smoothness, and angle-over-length (for all, lower value is better). We also introduce our custom metric, "safety distance" (minimum value of the SDF). The idea of smoothness metric is to look at the triangles formed by consecutive path segments and compute the angle between those segments. Then, the outside angle for the computed angle is normalized by the path segments and contributes to the path smoothness.

$$Smoothness = \sum_{i=2}^{n-1} \left(\frac{2\left(\pi - \arccos\left(\frac{a_i^2 + b_i^2 - c_i^2}{2a_i b_i}\right)\right)}{a_i + b_i} \right)^2, \quad (5)$$

where $a_i = dist(s_{i-2}, s_{i-1})$, $b_i = dist(s_{i-1}, s_i)$, $c_i = dist(s_{i-2}, s_i)$, s_i is the i^{th} state along the path and $dist(s_i, s_j)$ gives the distance between two states. AOL is an alternative measure of smoothness which divides the total heading change by the path length[54].

$$AOL = \frac{1}{l} \sum_{i=1}^{n-1} (|\theta_{i+1} - \theta_i|),$$
(6)

Planner	Time, s	Length, m	Smooth- ness	AOL	Safety distance, m
RRT*	11	2.27	0.008	0.005	0.048
RRT	0.013	2.72	0.012	0.010	0.148
InformedRRT	11	2.27	0.006	0.004	0.041
SBL	0.062	4.99	0.055	0.042	0.049
RRT+GRIPS	0.013	2.44	0.009	0.004	0.151
θ *+NPField (ours)	0.063	2.33	0.002	0.006	0.116

TABLE II COMPARATIVE STUDIES ON BENCHMR SCENARIOS

where θ_i is the heading angle of the *i*th path segment, *l* is the total length of the path.

The results are given in table II. We compare our stack (Theta* + NPField) with state of the art planners: RRT [18], RRT* [55], Informed RRT [56], SBL [57] and RRT with GRIPS [58] smoothing. We do not provide the results for PRM[19], Theta* with CIAO [1] optimization, and Theta* with CHOMP [8] optimization as they were able to generate a successful plan for less than a half of tasks. This result is particularly important for Theta* + CIAO and Theta* + CHOMP, as they are optimization-based planers similar to our approach and use the same global plans. However, they could not handle considered scenarios due to collisions (CHOMP) or failure to find a result (CIAO). Results in the table show that our stack is generally comparable to other planners. It provides nearly the shortest path length, the best smoothness, a good AOL, a good computation time, and a good safety distance. We cannot specify an approach, which is definitely better than ours (RRT with GRIPS is fast and safe but provides less smooth trajectories).

C. Real Robot Experiments

We deploy our approach on a real Husky UGV mobile manipulator as a ROS module for MPC local planning and control, which works with Theta* global planner. The testing scenario includes hat transportation through a twisty corridor. NPField model is able to reconstruct the repuslive potential of the actual obstacles on the map (see Fig. 5). The manipulator is holding the hat in an outstretched configuration (see Fig. 6). Therefore, a more complicated concave footprint is valid. Scenario execution may be seen in the accompanying video. During our experiment the robot succesfully completed a 15 meters path, which included three twists and one narrow passage.

VI. CONCLUSIONS

We propose a novel approach to local trajectory planning, where a Model Predictive Controller uses the neural model to estimate collision danger as a differentiable function. Our NPField neural architecture consists of encoders and NPFunction blocks. Encoders provide a compact representation of the obstacle map and robot footprint; this compact representation is sent to the MPC solver as a vector of problem parameters. NPFunction is integrated into the

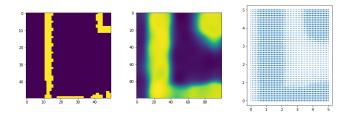


Fig. 5. Visualization of the potential field for the map of the real environment. Left: groundtruth map (obstacles marked with yellow); middle: predicted neural potential field for the vertical orientation of the robot; right: visualization of the algorithmically-calculated SDF for the vertical orientation of the robot.



Fig. 6. Husky mobile manipulator transporting a hat. A footprint with an outstretched manipulator is valid for this task.

optimization loop, and its gradients are used for trajectory correction. We implement our controller using Acados MPC framework and L4CasADi tool for integrating deep neural models into MPC loop. Our approach allows the robot with a complicated footprint to successfully navigate among the obstacles in real time. A planning stack Theta* + NPField showed comparable results with sample-based planners on the BenchMR testing framework.

We consider our work a starting point for further research on neural potential estimation for kinodynamic planning for various robotic systems in various environments. Trajectory planning on more complex maps (e.g. elevation maps) is a promising topic of the future research. Another important aspect is further research on footprint encoding, which may be useful for planning the trajectories of the robotic system with changing footprints (e.g. mobile manipulators under whole-body control).

REFERENCES

- Tobias Schoels et al. "An NMPC Approach using Convex Inner Approximations for Online Motion Planning with Guaranteed Collision Avoidance". In: 2020 IEEE International Conference on Robotics and Automation (ICRA). 2020, pp. 3574–3580. DOI: 10.1109/ ICRA40945.2020.9197206.
- [2] Damir Bojadžić et al. "Non-holonomic RRT & MPC: Path and trajectory planning for an autonomous cycle rickshaw". In: arXiv preprint arXiv:2103.06141 (2021).
- [3] Zhiqiang Zuo et al. "MPC-based cooperative control strategy of path planning and trajectory tracking for intelligent vehicles". In: *IEEE Transactions on Intelligent Vehicles* 6.3 (2020), pp. 513–522.

- [4] Jie Ji et al. "Path planning and tracking for vehicle collision avoidance based on model predictive control with multiconstraints". In: *IEEE Transactions on Vehicular Technology* 66.2 (2016), pp. 952– 964.
- [5] Jun Zeng, Bike Zhang, and Koushil Sreenath. "Safety-critical model predictive control with discrete-time control barrier function". In: 2021 American Control Conference (ACC). IEEE. 2021, pp. 3882– 3889.
- [6] Lars Blackmore, Masahiro Ono, and Brian C Williams. "Chanceconstrained optimal path planning with obstacles". In: *IEEE Transactions on Robotics* 27.6 (2011), pp. 1080–1094.
- [7] Akshay Thirugnanam, Jun Zeng, and Koushil Sreenath. "Safety-Critical Control and Planning for Obstacle Avoidance between Polytopes with Control Barrier Functions". In: 2022 International Conference on Robotics and Automation (ICRA). 2022, pp. 286–292. DOI: 10.1109/ICRA46639.2022.9812334.
- [8] John Schulman et al. "Motion planning with sequential convex optimization and convex collision checking". In: *The International Journal of Robotics Research* 33 (2014), pp. 1251–1270.
- [9] O. Khatib. "Real-time obstacle avoidance for manipulators and mobile robots". In: *Proceedings. 1985 IEEE International Conference on Robotics and Automation.* Vol. 2. 1985, pp. 500–505. DOI: 10. 1109/ROBOT.1985.1087247.
- [10] Tobias Schoels et al. "CIAO*: MPC-based Safe Motion Planning in Predictable Dynamic Environments". In: *IFAC-PapersOnLine* 53.2 (2020). 21st IFAC World Congress, pp. 6555–6562. ISSN: 2405-8963. DOI: https://doi.org/10.1016/j.ifacol. 2020.12.072. URL: https://www.sciencedirect.com/ science/article/pii/S2405896320303281.
- [11] Ben Mildenhall et al. NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis. 2020. arXiv: 2003.08934 [CS.CV].
- [12] Daniil Kirilenko et al. "TransPath: Learning Heuristics for Grid-Based Pathfinding via Transformers". In: Proceedings of the AAAI Conference on Artificial Intelligence 37.10 (June 2023), pp. 12436– 12443. DOI: 10.1609/aaai.v37i10.26465. URL: https: //ojs.aaai.org/index.php/AAAI/article/view/ 26465.
- [13] Samuel Triest et al. "Learning Risk-Aware Costmaps via Inverse Reinforcement Learning for Off-Road Navigation". In: arXiv preprint arXiv:2302.00134 (2023).
- [14] Mateo Guaman Castro et al. "How does it feel? self-supervised costmap learning for off-road vehicle traversability". In: 2023 IEEE International Conference on Robotics and Automation (ICRA). IEEE. 2023, pp. 931–938.
- [15] David González et al. "A Review of Motion Planning Techniques for Automated Vehicles". In: *IEEE Transactions on Intelligent Transportation Systems* 17.4 (2016), pp. 1135–1145. DOI: 10.1109/ TITS.2015.2498841.
- [16] Peter E. Hart, Nils J. Nilsson, and Bertram Raphael. "A Formal Basis for the Heuristic Determination of Minimum Cost Paths". In: *IEEE Transactions on Systems Science and Cybernetics* 4.2 (1968), pp. 100–107. DOI: 10.1109/TSSC.1968.300136.
- [17] Zain Alabedeen Ali and Konstantin Yakovlev. "Safe interval path planning with kinodynamic constraints". In: *Proceedings of the 37th AAAI Conference on Artificial Intelligence (AAAI 2023)*. 2023, pp. 12330–12337.
- [18] Steven M. LaValle and Jr. James J. Kuffner. "Randomized Kinodynamic Planning". In: *The International Journal of Robotics Research* 20.5 (2001), pp. 378–400. DOI: 10.1177 / 02783640122067453.
- [19] L.E. Kavraki et al. "Probabilistic roadmaps for path planning in high-dimensional configuration spaces". In: *IEEE Transactions on Robotics and Automation* 12.4 (1996), pp. 566–580. DOI: 10. 1109/70.508439.
- [20] Brian Angulo, Aleksandr Panov, and Konstantin Yakovlev. "Policy Optimization to Learn Adaptive Motion Primitives in Path Planning With Dynamic Obstacles". In: IEEE Robotics and Automation Letters 8.2 (2023), pp. 824–831. ISSN: 2377-3766. DOI: 10.1109/LRA. 2022.3233261. URL: https://ieeexplore.ieee.org/ document/10003648/.
- [21] Alexey Skrynnik et al. "Pathfinding in stochastic environments: learning vs planning". In: *PeerJ Computer Science* 8 (2022), e1056. ISSN: 2376-5992. DOI: 10.7717/peerj-cs.1056. URL: https://peerj.com/articles/cs-1056.

- [22] Dong Hun Kim and Seiichi Shin. "Local path planning using a new artificial potential function composition and its analytical design guidelines". In: Advanced Robotics 20 (2006), pp. 115–135.
- [23] Jing Ren, K.A. McIsaac, and R.V. Patel. "Modified Newton's method applied to potential field-based navigation for mobile robots". In: *IEEE Transactions on Robotics* 22.2 (2006), pp. 384–391. DOI: 10. 1109/TRO.2006.870668.
- [24] Rafal Szczepanski, Tomasz Tarczewski, and Krystian Erwinski. "Energy Efficient Local Path Planning Algorithm Based on Predictive Artificial Potential Field". In: *IEEE Access* 10 (2022), pp. 39729– 39742. DOI: 10.1109/ACCESS.2022.3166632.
- [25] Elmer G Gilbert, Daniel W Johnson, and S Sathiya Keerthi. "A fast procedure for computing the distance between complex objects in three-dimensional space". In: *IEEE Journal on Robotics and Automation* 4.2 (1988), pp. 193–203.
- [26] Simon Zimmermann et al. "Differentiable collision avoidance using collision primitives". In: 2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE. 2022, pp. 8086–8093.
- [27] Zeqing Zhang et al. "A generalized continuous collision detection framework of polynomial trajectory for mobile robots in cluttered environments". In: *IEEE Robotics and Automation Letters* 7.4 (2022), pp. 9810–9817.
- [28] Julius Ziegler et al. "Trajectory planning for Bertha A local, continuous method". In: 2014 IEEE Intelligent Vehicles Symposium Proceedings. 2014, pp. 450–457. DOI: 10.1109/IVS.2014. 6856581.
- [29] Mrinal Kalakrishnan et al. "STOMP: Stochastic trajectory optimization for motion planning". In: May 2011, pp. 4569–4574. DOI: 10. 1109/ICRA.2011.5980280.
- [30] Grady Williams et al. "Aggressive driving with model predictive path integral control". In: 2016 IEEE International Conference on Robotics and Automation (ICRA). 2016, pp. 1433–1440.
- [31] Grady Williams et al. "Information theoretic MPC for model-based reinforcement learning". In: 2017 IEEE International Conference on Robotics and Automation (ICRA). IEEE. 2017, pp. 1714–1721.
- [32] Charles Dawson, Sicun Gao, and Chuchu Fan. "Safe control with learned certificates: A survey of neural lyapunov, barrier, and contraction methods for robotics and control". In: *IEEE Transactions on Robotics* (2023).
- [33] Michal Adamkiewicz et al. "Vision-Only Robot Navigation in a Neural Radiance World". In: *IEEE Robotics and Automation Letters* 7.2 (2022), pp. 4606–4613. DOI: 10.1109/LRA.2022.3150497.
- [34] Hossein Abdi, Golnaz Raja, and Reza Ghabcheloo. "Safe Control using Vision-based Control Barrier Function (V-CBF)". In: 2023 IEEE International Conference on Robotics and Automation (ICRA). IEEE. 2023, pp. 782–788.
- [35] Yeseung Kim, Jinwoo Kim, and Daehyung Park. "GraphDistNet: A Graph-Based Collision-Distance Estimator for Gradient-Based Trajectory Optimization". In: *IEEE Robotics and Automation Letters* 7.4 (2022), pp. 11118–11125.
- [36] Mikhail Kurenkov et al. "NFOMP: Neural Field for Optimal Motion Planner of Differential Drive Robots With Nonholonomic Constraints". In: *IEEE Robotics and Automation Letters* 7.4 (2022), pp. 10991–10998. DOI: 10.1109/LRA.2022.3196886.
- [37] Ali Punjani and Pieter Abbeel. "Deep learning helicopter dynamics models". In: 2015 IEEE International Conference on Robotics and Automation (ICRA). 2015, pp. 3223–3230. DOI: 10.1109/ICRA. 2015.7139643.
- [38] Alessandro Saviolo, Guanrui Li, and Giuseppe Loianno. "Physics-Inspired Temporal Learning of Quadrotor Dynamics for Accurate Model Predictive Trajectory Tracking". In: *IEEE Robotics and Automation Letters* 7.4 (2022), pp. 10256–10263. DOI: 10.1109/ LRA.2022.3192609.
- [39] Nathan A. Spielberg, Matthew Brown, and J. Christian Gerdes. "Neural Network Model Predictive Motion Control Applied to Automated Driving With Unknown Friction". In: *IEEE Transactions on Control Systems Technology* 30.5 (2022), pp. 1934–1945. DOI: 10.1109/ TCST.2021.3130225.
- [40] Kong Yao Chee, Tom Z. Jiahao, and M. Ani Hsieh. "KNODE-MPC: A Knowledge-Based Data-Driven Predictive Control Framework for Aerial Robots". In: *IEEE Robotics and Automation Letters* 7.2 (2022), pp. 2819–2826. DOI: 10.1109/LRA.2022.3144787.
- [41] Taekyung Kim et al. "TOAST: Trajectory Optimization and Simultaneous Tracking Using Shared Neural Network Dynamics". In: *IEEE Robotics and Automation Letters* 7.4 (2022), pp. 9747–9754.

- [42] Tim Salzmann et al. "Real-Time Neural MPC: Deep Learning Model Predictive Control for Quadrotors and Agile Robotic Platforms". In: *IEEE Robotics and Automation Letters* 8.4 (2023), pp. 2397–2404. DOI: 10.1109/LRA.2023.3246839.
- [43] Robin Verschueren et al. Acados: a modular open-source framework for fast embedded optimal control. 2020. arXiv: 1910.13753 [math.OC].
- [44] Tim Salzmann. TUM-AAS/ml-casadi: Use PyTorch Models with CasADi and Acados. 2023. URL: https://github.com/TUM-AAS/ml-casadi.
- [45] Avadesh Meduri et al. "MPC with Sensor-Based Online Cost Adaptation". In: 2023 IEEE International Conference on Robotics and Automation (ICRA). IEEE. 2023, pp. 996–1002.
- [46] Mateja Novak, Tomislav Dragicevic, and Frede Blaabjerg. "Weighting factor design based on Artificial Neural Network for Finite Set MPC operated 3L-NPC converter". In: 2019 IEEE Applied Power Electronics Conference and Exposition (APEC). IEEE. 2019, pp. 77–82.
- [47] Xin Wang et al. "Neural network based weighting factor selection of mpc for optimal battery and load management in mea". In: 2020 23rd International Conference on Electrical Machines and Systems (ICEMS). IEEE. 2020, pp. 1763–1768.
- [48] Ashish Vaswani et al. Attention Is All You Need. 2023. arXiv: 1706. 03762 [cs.CL].
- [49] Alexey Dosovitskiy et al. An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale. 2021. arXiv: 2010.11929 [CS.CV].
- [50] N. Sturtevant. "Benchmarks for Grid-Based Pathfinding". In: Transactions on Computational Intelligence and AI in Games 4.2 (2012), pp. 144–148. URL: http://web.cs.du.edu/~sturtevant/ papers/benchmarks.pdf.
- [51] Joel A E Andersson et al. "CasADi A software framework for nonlinear optimization and optimal control". In: *Mathematical Programming Computation* 11.1 (2019), pp. 1–36. DOI: 10.1007/ s12532-018-0139-4.
- [52] Tim Salzmann et al. "Learning for CasADi: Data-driven Models in Numerical Optimization". In: (2023). arXiv: 2312.05873.
- [53] Alex Nash et al. "Theta^{*}: Any-angle path planning on grids". In: AAAI. Vol. 7. 2007, pp. 1177–1183.
- [54] Eric Heiden et al. "Bench-MR: A Motion Planning Benchmark for Wheeled Mobile Robots". In: *IEEE Robotics and Automation Letters* 6.3 (2021), pp. 4536–4543. DOI: 10.1109/LRA.2021. 3068913.
- [55] Sertac Karaman and Emilio Frazzoli. "Sampling-based algorithms for optimal motion planning with deterministic μ-calculus specifications". In: 2012 American Control Conference (ACC). IEEE. 2012, pp. 735–742.
- [56] Jonathan D Gammell, Siddhartha S Srinivasa, and Timothy D Barfoot. "Informed RRT: Optimal sampling-based path planning focused via direct sampling of an admissible ellipsoidal heuristic". In: 2014 IEEE/RSJ international conference on intelligent robots and systems. IEEE. 2014, pp. 2997–3004.
- [57] David Hsu, J-C Latombe, and Rajeev Motwani. "Path planning in expansive configuration spaces". In: *Proceedings of international conference on robotics and automation*. Vol. 3. IEEE. 1997, pp. 2719–2726.
- [58] Eric Heiden et al. "Gradient-informed path smoothing for wheeled mobile robots". In: 2018 IEEE International Conference on Robotics and Automation (ICRA). IEEE. 2018, pp. 1710–1717.